Agent-Based Models and Behavioral Operational Research

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ABSTRACT

This chapter sets out agent-based modelling as a promising methodology for behavioural operational research. We set out the links between existing modelling techniques such as system dynamics and discrete event simulation, and offer examples of how agent-based models can be used to model the behavior of individuals. We show how existing system-level models can be 'agentized' so that system-level behavior is modelled by the interactions of individual agents. This focus on the individuals in the system rather than the system itself opens up a rich prospectus for the use of agent-based modelling within behavioural operational research.

BIOGRAPHY

Dr Duncan Robertson is a member of the management sciences and operations management group at Loughborough University, previously having worked at Warwick Business School, Manchester Business School, Sun Yat-sen University, and was a researcher at The Wharton School. He is a Fellow of St Catherine's College, Oxford. Before undertaking his DPhil at Saïd Business School, he qualified as a Chartered Accountant with KPMG. He originally trained as a physicist at Imperial College London.

Chapter 7

Agent-Based Models and Behavioral Operational Research

Duncan A. Robertson

7.1 Introduction

Agent-Based Modelling is a form of computational modelling where individual constituent components of a system – such as individuals, firms, cells, or atoms – are modelled. In Behavioral Operational Research, these constituent *agents* can be people within a group of interest, but can also include interactions with members in other groups or interactions with the wider environment in which agents reside. Agents within the system may act according to simple rules or heuristics, which gives rise to the interactions between these agents. These interactions can combine in such a way that emergent properties are seen – properties that are not imposed on the model in a top-down fashion but are generated by the interactions of agents themselves. Agent-Based Modelling is not however confined to systems that exhibit equilibrium, but can be used to model the dynamics of a system over time. As such, Agent-Based Modelling provides a rich simulation methodology to augment and potentially extend more traditional modelling techniques, where behavior and interactions of individuals are at the forefront of the modeler's mind.

Sterman (1989) highlights in the context of systems dynamics models such as the Beer Distribution Game (Forrester, 1961), the micro-behavior of actors within the system generates dynamics, such as the bullwhip effect, where boundedly rational (Simon, 1957) behavior by the individuals, particularly their "misperceptions of feedback" generate dynamics that are unpredictable and are contrary to purely rational behavior. Sterman states that merely understanding the behavior of individuals and carrying out observations of individuals is not sufficient: what is needed is to "understand how micro-level behaviors link to the behavior of the system" (Coleman 1987).

We follow and extend Sterman's (1989) approach to models that, due to the complexity of the behavioral interactions between individuals within the system, cannot successfully be modelled using systems dynamics or systems-level modelling, but whose interaction at the micro level *forms* the system – behavior that is *emergent* (Goldstein, 1999) and that follows macro-behavior from micro-behavioral interactions (Schelling, 1978). Without understanding and modeling the behavioral characteristics of the agents, we are very much restricted in our understanding from the model if we intend to model the system from purely a systems perspective.

Hämäläinen *et al.* (2013) extends this to a call for the development of a holistic field of Behavioral Operational Research, extending Cronin *et al.*'s (2009)'s work showing that system-level constructs such as the concept of accumulation are not well understood by otherwise intelligent, rational, individuals. Hämäläinen and Saarinen (2006) and Saarinen and Hämäläinen (2007) advocate educating individuals in "systems intelligence" allowing users to sense the "feeling of the system" (Hämäläinen and Saarinen, 2008) to overcome Ackoff's (2006) assertion that relatively few organizations adopt systems thinking.

We will discuss some of the reasons for this: the systems that we profess to understand are *complex systems* of interacting individuals and as such it is not truly possible to understand the emergent, system-level properties that we tend to interpret without understanding the micro-level interactions that make up that emergent system – for example the actors that are making up the accumulation in Cronin *et al.* (2009) and Hämäläinen *et al.*'s (2013) work: in system-level models, we cannot visualize the individuals and the system is therefore too abstract for "well educated adults" (Cronin *et al.*, 2009) to comprehend. We will advocate the use of agent-based models to explore these micro-level interactions whose

Behavioral interactions, either between actors or between individual actors and the system itself, contribute to or indeed *are*, the behavior of the system itself.

7.2 Complex Systems of Interacting Individuals

7.2.1 Complex Systems

Complexity science is engaged with the understanding of systems comprising of interacting agents, for example the interactions between individuals in a social system. Johnson (2009) acknowledges that many of the systems that we currently analyze may be thought of as complex systems without being studied explicitly as such: "however, the *way* in which scientists have traditionally looked at these systems does not use any of the insight of Complexity Science". The same is true of Behavioral Operational Research: we are so used to studying the system as a whole, we have neglected individual behavior. One definition of complexity science is "the study of the phenomena which emerge from a collection of interacting objects" (Johnson, 2009). In our case, these interacting objects are individual people, and the phenomena are the socially constructed behavior of the system itself.

We shall concentrate on one insight from Complexity Science, that of studying the agents that comprise a system, that is to say that interactions between agents within a system, interactions between agents and the environment, and interactions between participants studying the system behavior for example in a workshop environment.

We agree that wholeheartedly with Luoma *et al.* (2010) that so-called "complex responsive process", or what we would describe more broadly as complex systems, should be integrated with systems thinking, and not seen as a rival theory. Complex Systems are broader than "complex responsive processes", and are applied to a wide range of systems, some deterministic, and some social. These systems are *complex* in that the system is made up of the individual parts of the system, and essentially the interactions between these individuals.

Agents within the system interact with each other but *also with the system itself.* We will show through introducing a range of agent-based models that, by looking at the system from the bottom-up, we can *generate* systems behavior which in turn can be sensed by the individual actors (with appropriate levels of systems intelligence) and their behavior can adapt not only to the local interaction but also with the system itself.

There are three levels of interaction that we can explore within these complex systems of interacting agents: behavioral interactions between agents, behavioral interactions between agents and the environment, and interactions between model users and the model itself.

7.2.2 Agent-Based Modelling

Agent-Based Modelling is a relatively recent approach to modelling systems of interacting individuals. Originally called Individual-Based Modeling (Hiebeler, 1994; Grimm and Railsbark, 1997), these models concentrate on the behavior of the components of the system rather than the system itself. The system behavior results from the micro-level interactions between the agents and, while a central policy maker is not required, policy-like or *emergent* behaviors of the system can result from the interactions between individuals.

Agent-based models are typically conceived using simple behaviors of individual agents (which can be made more complicated or refined as the model is developed). These behaviors are boundedly rational and are based on the agent's own perception of its environment – perspectives that can differ between different agents. Typically, heuristics or simple rules are used to model behavior. But there is nothing in the philosophy of agent-based modeling that requires simplicity of behavior: intricate decision rules can easily be modelled and studied. The advantage of agent-based approaches is that complexity of behavior can be increased, by sequentially turning on more and more complicated behavioral rules, in order to determine what behavioral characterisitics are required for a system to change state.

Agents within an agent-based models are autonomous, in that each agent is individually modeled as different objects within the simulation. Each time interval, step, or 'tick' of the model results in agents obeserving their environment (including interactions with other agents), undertaking an action or movement based on decision rules or heuristics, and the system being updated as a result of all the individual movements of the agents.

It is worth noting that agent-based approaches are flourishing in behavioral finance (*Economist*, 2010; Farmer and Foley, 2009) and in behavioral economics (Tesfatsion, 2002; 2006). We introduce below several models that focus on the interactions between agents.

Figure 7.1 shows how traditional models can be 'agentized' in order to include behavioral effects that may be ignored or averaged out in traditional, analytical models, and how some agent-based models have no equivalent in system-level models (shown by the question mark).

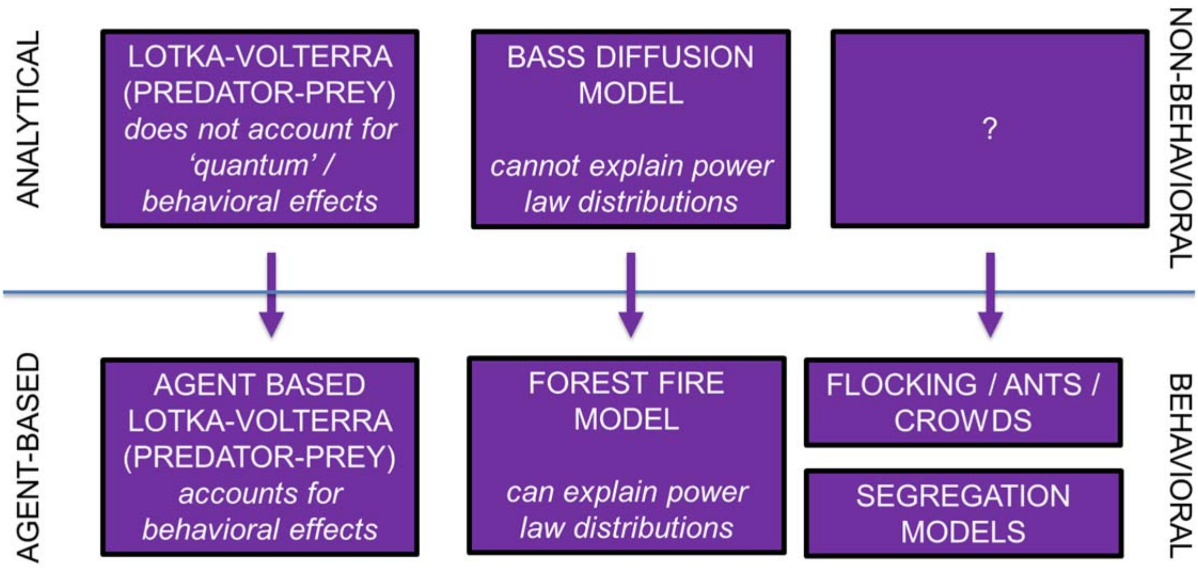


Figure 7.1: Agent-Based vs. Analytical Modeling Approaches

Agent-Based Modelling can be differentiated from system dynamics modelling and discrete event simulation (Borshchev and Filippov 2004, Robinson 2014). System dynamics modelling focuses on stocks and flows, concentrating on feedback loops and time delays that link these stocks. These feedback loops may compound or retard eachother, in that they may be "reinforcing" or "balancing". *System Dynamics* models abstract away the individual entities that flow within the system, preferring to quantify these as a level of flow: individual entities are quantified rather than being modelled explicitly. A side effect of this is that the entities that comprise the stocks and flows are treated as being *fungible* – their individuality is abstracted away. The interactions between stocks, flows, and delays, can be shown on a system dynamics diagram which is a way of visually representing the differential equations that fully describe the model. While system dynamics models are continous, in that they evolve over time by means of differential equations, discrete event simulation models are discrete. *Discrete-Event Simulation* relies on modelling the flow of entities through a system from one 'activity' to another, for example passengers flowing through an airport. As the activities (checkin, security etc.) do not clear at the same rate as the arrival of entities in to the system, queues form and are studied. Changes can be made to the configuration of activities and the way that entities are allowed to travel through the system, and these configurations can be compared. Discrete event simulation in comparison with system dynamics models can be modelled stochastically, in that events can occur randomly. *Agent-Based Modelling* can deal both with stochastic events and heterogeneity of entitites, or at the opposite extreme, we can model deterministic events with a homogeneous population of agents. Potentially, agent-based model can deal with more specific and idionsyncratic behaviors than discrete-event simulation or system dynamics.

7.3 Introducing Behavior to Existing Modelling Techniques

We will introduce three ways that agent-based modelling can incorporate behavioral aspects into the modelling repertoire. We shall see that even with homogeneous agents, behavior within the system produces interesting social- or system-level behavior. By opening up the ability to model interacting individuals, we open up a plethora of possibilities for explicitly studying how the behavior of individual agents changes the system as a whole.

The models are presented in order of increasing heterogeneity (and therefore behavioral characteristics) of the agents. Each model is spatial in that it locates agents with a set of coordinates in a space.

- In the first model, the Segregation model, agents are homogeneous in that they each have the same model-level parameter, being the proportion of their neighbors of a different type/color that they are willing to tolerate.
- The second model, the Predator-Prey model, introduces energy levels to the agents – when agents' energy is exhausted, they die.
- The third model, the Forest Fire model, has individuals in one of two states, activated (on fire) or prone to activation.

These models introduce the ability of agent-based models to incorporate the behaviors of individual actors.

Although the actors within the models as initially presented are not nuanced individuals exhibiting sophisticated behaviors (they are respectively, "white" or "grey" people; wolves or sheep, and perhaps most esoterically, trees!), these are merely the building blocks *from which* sophisticated behavioral models can be created. Without understanding the behaviors of models with simple rules, the complex interactions of more sophisticated models may confound our results. It is therefore important to start with the *simplest* model that exhibits interesting (tipping points, power laws, quantized) behavior and not to start from the most complicated system that we can imagine. The following are all dynamic models where behavioral interactions, either between individuals or between individuals and the environment, are critical in constructing the system, rather than modelling the macro-level system itself.

7.3.1 Tipping Points from Individual Behavior: Segregation Models

Schelling's (1969, 1971) model of segregation was one of the first models to approach a modelling question not by looking at the macro-level dynamics of the system itself but rather to model the behavior of individual actors and from those micro-level interactions, aggregate their behavior into the macro-level properties of the system.

The model is simple: individuals possess only one characteristic of interest – their happiness, which is derived from their type/color and the type/color of those around them. Agents' happiness level is simply generated by dividing the number of neighbors of the same type/color as themselves by the total number of neighbors. So, if a white agent is surrounded by 2 whites and 3 greys, their happiness level would be 2/5 = 0.4. If the agent's happiness level is below a system-level tolerance parameter, the agent is unhappy; if the agent's happiness level is equal to or greater than the tolerance parameter, the agent is happy. While agents are homogenous in the sense that they all have identical tolerance parameters, it is trivial to extend this to individual agents having heterogeneous tolerances. (Agents can possess other characteristics, such as shape which can be used to extend the model.)

(a) (b)

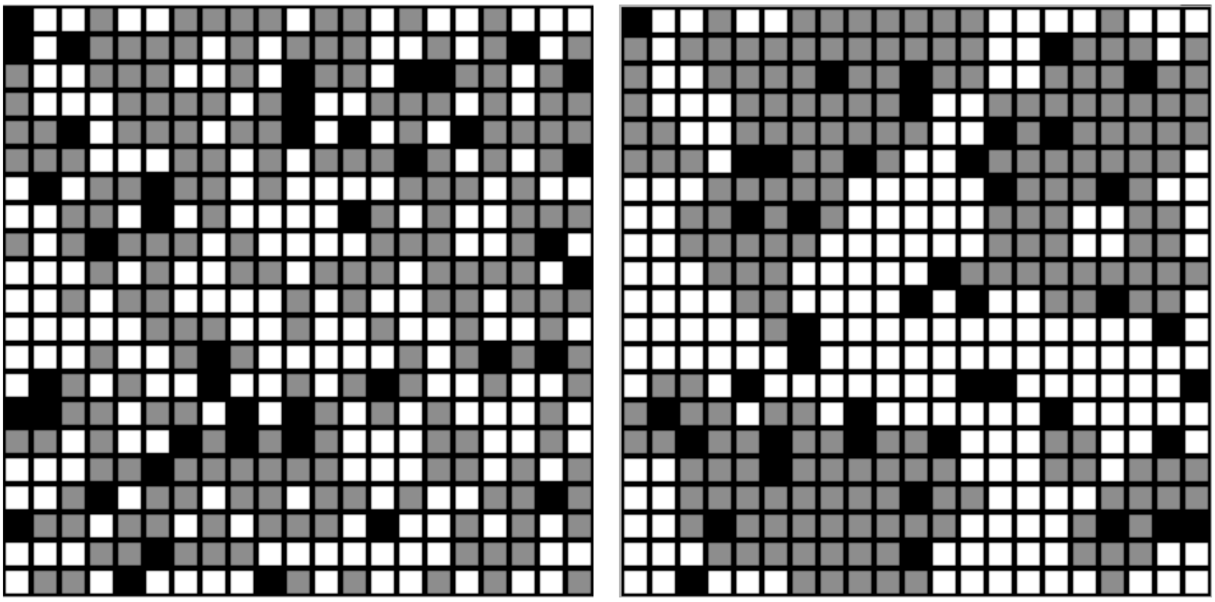


Figure 7.2: (a) Initial Conditions (b) Equilibrium Conditions showing segregation produced by agent-level behavior, not by the system itself

The model is constructed as follows:

- *N* agents are situated on a grid. *N*/2 are colored white, and *N*/2 are colored grey
- Each agent decides whether they are happy based on whether the percentage similar in their neighborhood is greater or equal to the model-level tolerance parameter
- Unhappy agents move to an unoccupied space
- The system updates until each agent is happy

The striking thing about this model is that it explains segregation not by individuals being overtly intolerant, but that preferences of only around 30% of neighbors to be the same type/color as the focal agent (meaning that you are happy in a 2:1 minority) results in segregation, as seen in Figure 7.3.

It is interesting to note that the system behavior, that of segregation, comes not from a policy of separating types/colors within a population, but from the emergent interactions between agents.

Agents in the segregation model do not need to be heterogeneous, in that each member of the population need not have the same, system-defined parameter of the proportion of agents of the same type/color that it wishes to have in its neighborhood. Of course, the model can easily be extended to give each agent specific behavior rules based on their own color, the behavior of neighboring agents etc., but this is not required to produce the interesting behavior exhibited by the model.

7.3.2 Individualizing Systems Models: Predator-Prey Models

Systems dynamics models by definition examine the state and changes to a system. Yet this system is comprised of interacting agents whose individual behavior aggregates to the behavior of the system.

The Lotka-Volterra population dynamics equations (Lotka 1920, 1925; Volterra 1926) are as follows:

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad (7.1)$$

$$\frac{dy}{dt} = -\gamma y + \delta xy \qquad (7.2)$$

where x and y are the population of prey and predators respectively, and α, β, γ and δ are parameters of the system representing the interaction of predators and prey.

The Lotka-Volterra model has been used widely within management science for modelling human systems as diverse as economic cycles (Goodwin, 1967), stock markets (Lee *et al.*, 2005), and the battle between old and new technologies such as the fountain pen and the ballpoint pen (Modis, 2003). All of these systems comprise interacting agents, yet these individual interactions are aggregated in the Lotka-Volterra population model into system-level dynamics.

While the differential equations linking the population of prey and predator are undoubtedly elegant, they overlook an important aspect of the system: the fact that the number of prey and predators is not a continuous variable: the values of x and y are in fact discrete. This matters where the number of individuals that have the potential to interact is low: there is no such thing as a part of predator or a prey interacting: either one predator interacts or no predators interact.

In order to convert the system-level model into a model where the interactions are of critical importance, we can personalize or *agentize* the model – in other words, each of the integer x's and y's (the quantity of prey and predators) are modelled as individual, autonomous agents.

This can be thought of as being analogous to a Kuhnian transition (Kuhn, 1962) between viewing a physical system through the models of Newtonian mechanics to alternative modeling techniques such as statistical mechanics or quantum mechanics. It is precisely this transition from treating the system as one entity to treating the system as comprised of interacting agents that Behavioral Operational Research is now facing: system level approaches are appropriate only for a certain class of problems at a certain scale. To use a physical analogy, individual components of the system, for example atoms within a lump of uranium can be averaged out and the macroscopic entity can be considered as one unit: if we throw a piece of uranium ore, we do not require any knowledge of quantum mechanics to predict its path. However, in order to understand the same uranium's behavior at a micro scale, we will require to view each atom separately using a different approach – that of quantum mechanics. Or indeed, if we are examining multiple interacting objects such as atoms within a gas, we can move to system-level characteristics of statistical mechanics where we ignore each individual atom and study the system as a whole. And we must note where Behavioral Operational Research can contribute most not in the regime of order (where there is correlation in behaviors of individuals where traditional methodologies work best), nor in the chaotic regime (where the system is so complicated that we have no hope of understanding all behavioral interactions). Behavioral Operational Research can give most in this center ground, that of complexity, where individual interactions are critical in determining how the system will behave.

Behavioral systems of interacting people *can* be modelled on the macro level by systems approaches, but this neglects potentially critical interactions between the actors that may be of vital importance. On one level, this may not matter, but on another, it may. A challenge for the field is to determine when individual behavioral interactions matter and when they can be averaged out and essentially ignored.

Order parameters (for a review see Sethna, 2006) are a potential solution to determine when a system is in the state when behavioral interactions can be ignored and the state where behavioral interactions cannot be ignored, where the symmetry and homogeneity of individuals can no longer be assumed.

By instead modelling individual behavior within the model (Figure 7.3) we can identify individuals within the system – in this case, wolves and sheep (predators and prey) but they could as easily be individuals who are using old and new technologies.

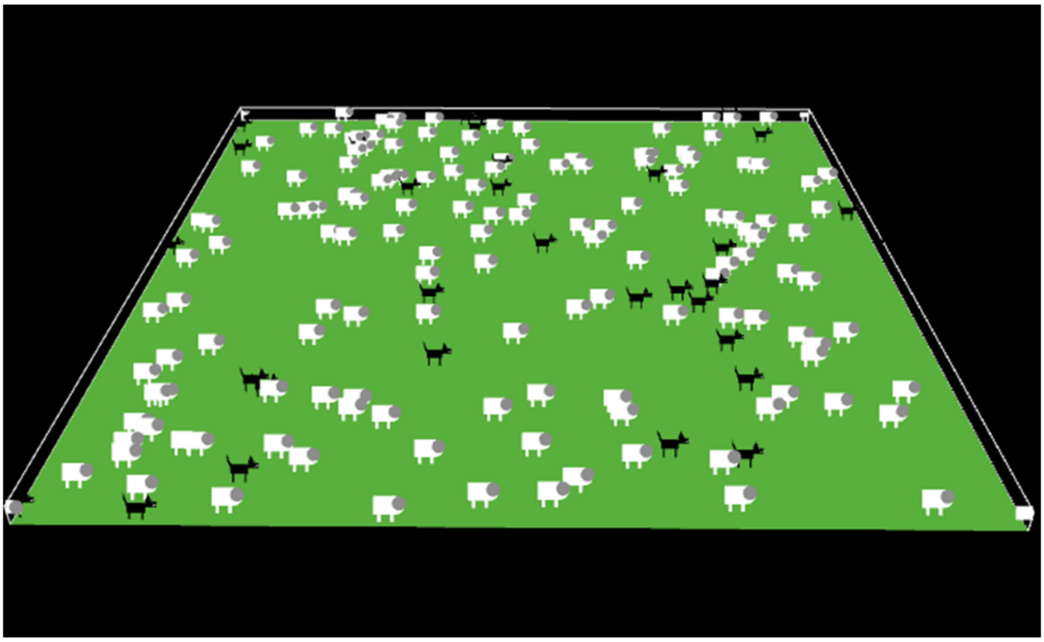


Figure 7.3: Agent-Based Operationalization of the Lotka-Volterra Systems Model

The rules/heuristics for *agentizing* the Lotka-Volterra model are relatively simple:

- Populate the space with x predators and y prey. Give each agent (predator or prey) a random quantity of energy. Note that x and y are integers as the number of agents is quantized: each predator contributes exactly 1 to the value of x; each prey contributes exactly 1 to the value of y.
- Allow the agents to move, predate, die, and reproduce.
- Movement – movement is undertaken by both predator and prey by taking a step of a random walk (in any direction) each time period. Movement is costly for predators, so they lose a unit of energy every time they move.
- Predation – if predators find themselves co-located with a prey, they will eat the prey and an amount of energy is added to the individual predator's energy level.
- Death – remove each predator or prey whose energy level has reduced to (or below) zero

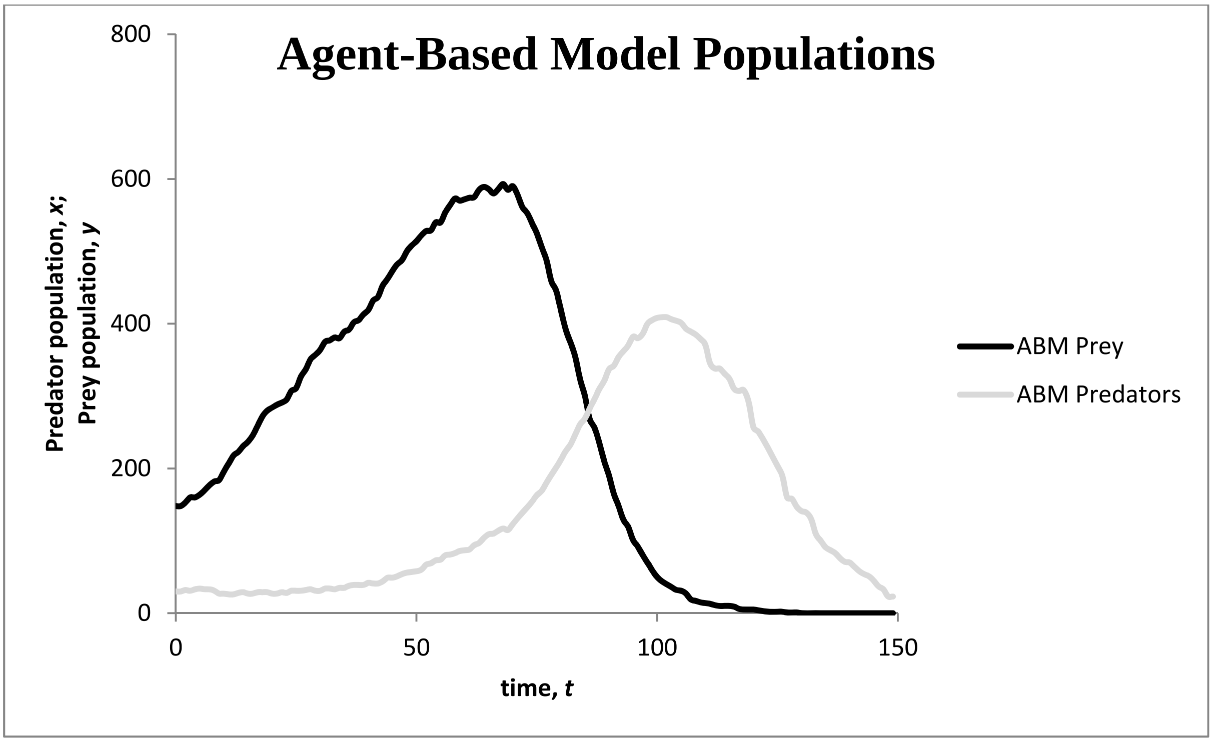
- Reproduction – predators reproduce with a fixed probability; prey reproduce with a fixed probability. The energy level of the parent is divided equally between the parent and the offspring.

When the agent model and the systems dynamics models are run side-by-side (Wilensky, 2005), we can see that the qualitative behavior of both systems (the systems dynamics model being the system itself, and the agent-based model being the aggregate behavior of the individual agents) are remarkably similar (See Figure 7.4). This process is called *docking* and is the alignment of different computational models (Axtell *et al.*, 1996).

By averaging out individual behavior in the systems dynamics model and thereby assuming homogenous agents produces an inferior model at the scale/order where individual behavior matters. It is in effect assuming statistical mechanics where the system reaches a thermodynamic limit (Hill, 1994).

Averaging out of individual behavior to produce a statistic of representative agent behavior below this thermodynamic- or order parameter- limit is not required and if done, can produce an inferior model.

(a)



(b)

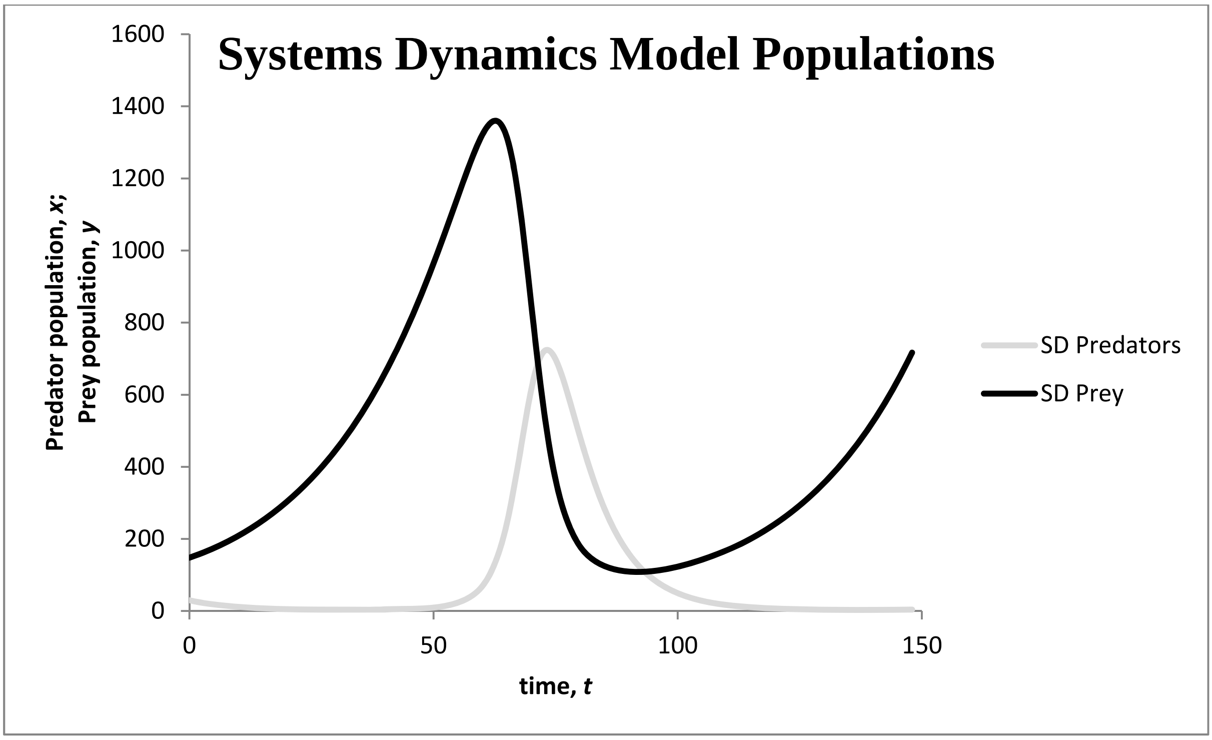


Figure 7.4: Population Comparisons between (a) Agent Based Model (ABM) implementation and (b) the original Systems Dynamics (SD) Lotka-Volterra Model

7.3.3 Power Laws: Forest Fire Models

The Forest Fire model (Bak *et al.*, 1990; Drossel and Schwabl, 1992) is a simple model of the interaction between agents. It is an interesting example of how the natural sciences make use of *toy models* where the phenomenon of interest (percolation) is studied by abstracting away behavior. As we shall see, we can think of the Forest Fire model as a social network model.

Power laws and Zipf laws (1949, 1935) can be seen in the distribution of firm sizes (Axtell, 2001), and were indeed studied by Herbert Simon (1955). It is also interesting to note that they were also studied by Lotka (1926) (of Lotka-Volterra laws, shown in our second agent-based model, above) in relation to scientific productions.

The Forest Fire model is based on a simple grid of interacting agents, in this case trees within a forest. The cells within the grid can be in any of three states: empty, occupied by a tree, or occupied by a tree that is on fire (see Figure 7.5).

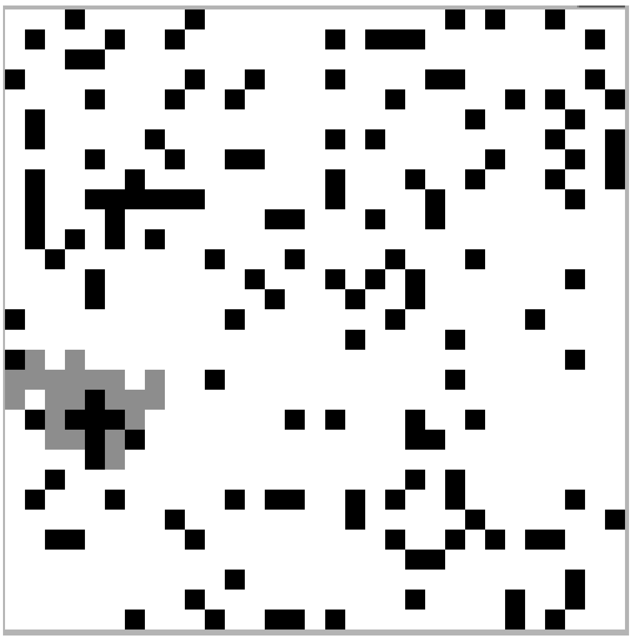


Figure 7.5: The Drossel and Schwabl (1992) Forest Fire Model (Operationalized in: Watts and Gilbert, 2014) showing blank cells (black) and the spread of fire (grey) through trees (white)

The rules of the Forest Fire model are as follows:

- empty cells turn into trees with probability *p*.
- a tree will catch fire with probability *f* : this is akin to lightning strikes within the forest.
- a tree will also catch fire if at least one of its neighbors is on fire.
- a burning tree will turn into an empty space.

Even though the Forest Fire model is one of a fire spreading through the forest, it can also be thought of as the basis for diffusion of ideas by virtue of word-of-mouth interactions. It can also be thought of as a constrained social network model where an individual can have up to four or eight neighbours (depending on whether a Von Neumann or a Moore neighbourhood is used). This can easily be extended to diffusion not on a grid but within a social network. Even though agents are represented on our grid, we can trivially transform this into a social network representation (see Figure 7.6).

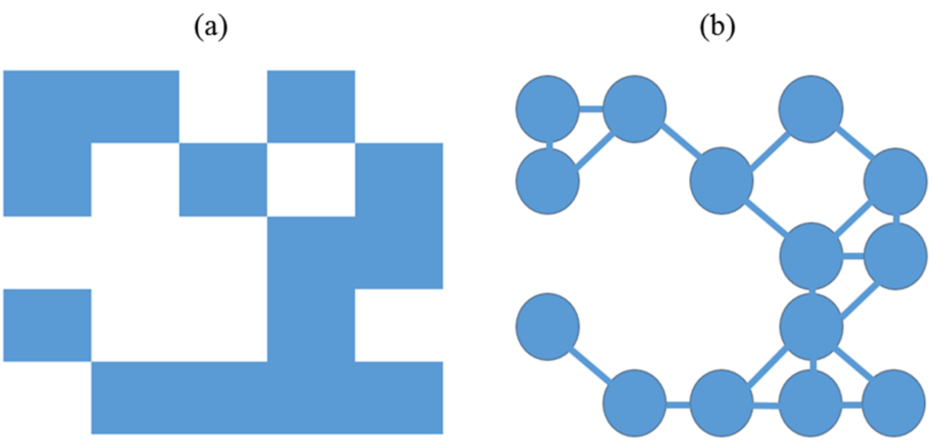


Figure 7.6: (a) Cellular and (b) Social Network Representations

When the Forest Fire model is run, a lightning strike (with probability set by parameter *f*) causes the spread of the fire throughout the forest spreading to contiguous areas of trees. The distribution of size of these fires in the Drossel and Schwabl (1992) model follows a power law distribution, meaning that large, catastrophic fires occur much less frequently than small fires. In the interpersonal world, this behavior has been found in distributions of sizes of riots. The Forest Fire model has also been used to augment the Bass (1969) diffusion model (Goldenberg *et al.*, 2001).

The Forest Fire model is interesting from a Behavioral Operational Research point of view as it models the spread of an idea at a micro level. The system effect that is observed is the size of the outbreak – which is a power law distribution (see Figure 7.7). This output cannot easily be modelled by conventional techniques.

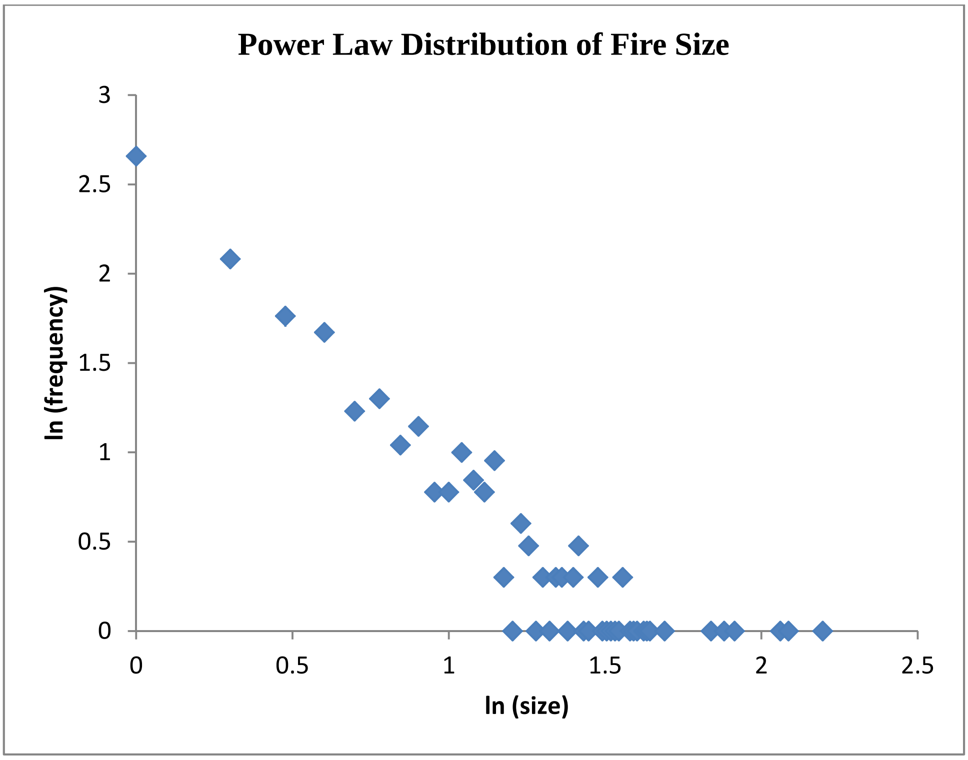


Figure 7.7: Sample Power Law of Distribution of Fire Size

The model also introduces the concept of a critical parameter – in the Forest Fire model of order *p*/*f*, where the behavior of the system changes from one regime to another.

Schelling's segregation model, Lotka-Volterra's predator-prey model, and Drossel-Schwabl's forest fire model are examples of how models from other disciplines can be used to create models that use individual characteristics other than ethnicity (in the segregation model) and individuals rather than animals (in the predator-prey model) and trees (in the forest fire model) to inform and create new models of behavioral operational research. A rich research agenda is opening up, which is outlined below.

7.4 A Research Agenda for Agent-Based Behavioral Operational Research

We have reviewed several approaches where agent-based modelling can be used to augment, and in certain circumstances improve, traditional system-level models.

Agent-based models are however not a panacea: there are circumstances where they undoubtedly unlock understanding of a system that would not be understood without them, for example when systems tip from one state to another.

It is important to note that we have introduced very simple models in this paper. This is deliberate. We are looking to introduce to the reader the simplest behavioral models that produce interesting results. Each of these models can be extended trivially to include more agent-specific parameters or behavioral assumptions. And this leads us to our first agenda item:

Which Behavioral Characteristics Matter?

A temptation of agent-based models as a modelling approach is to include every possible trait of behavior into the model. Sterman (1989) and Morecroft (1983, 1985) emphasize the effects of boundedly rational individual agents, yet analyze the system *at* the system level. Agent-based models allow us to start our models with the simplest agent behavior that produces interesting results. However, in this modelling process (which may be facilitated as discussed below), an agent-based model allows us to include variables, parameters, and behaviors that can be switched on or off as part of the modelling process. In this way, we can extend the model; this can be a particularly fruitful area of future research as part of a facilitated model building (see below).

Defining Order Parameters for Systems where Intra-Model Behavior is Important

While model – individual interactions will always be important, we can also define regimes where behavioral interactions *within* the model are important. We want to be able to restrict our behavioral work where behavioral interactions are actually important and change the model itself. We want to ignore regimes where systems are in a stable state and traditional models can be used; similarly, we want to avoid studying the behavioral implications of systems that are chaotic. We can define order parameters, for example the level of intolerance in the Schelling segregation model which determine on the level of individual interaction where the system transitions from one state to another. The middle, complex regime is where we should concentrate our attention in Behavioral Operational Research.

Quantized/Individual Behavior is Important: 'Agentization' of Models

Systems dynamics models ignore individuals and hence ignore individual behavior. The Lotka-Volterra model when 'agentized' produces qualitatively similar results but differs in that the system collapses in the agent-based model when the last agent is removed from the system. In a systems dynamics model, numbers of individuals below one can exist even this has no parallel in reality. Behavioral effects exist on the individual level and not on the system level. By studying existing models and moving them to individual-based models rather than system models is a rich avenue for future research.

Toy Models for Behavioral Operational Research: Agent-Based Facilitation

We can learn from the natural sciences in creating simplified models of interaction that can be used to understand a *different but similar system*. We can then alter the agents within the model, using feedback from psychological understanding of individuals or by feedback obtained by participants in the modeling process. In this way, we can experiment very quickly and efficiently with different behavioral heuristics.

Agent-based modelling has a clear opportunity to act as a part of facilitated model building (Franco and Montibeller, 2010; Franco and Rouwette, 2011; Rouwette 2011) where participants are able to interact with, construct, and adapt models. Recent advances in agent-based modelling software facilitates this, with newer software packages such as *NetLogo* creating huge opportunities to develop models on the fly rather than writing low-level code to manipulate agents in first generation agent-based modelling software such as *Swarm* and *RePast* (Robertson, 2005).

Agent-based models such as the ones introduced in this paper, when presented to audiences, inevitably result in contributions from the audience suggesting ways of making more detailed behavioral rules for participants. In short, we can use agent-based models rather than systems dynamics (as discussed in Rouwette *et al.*, 2002) or discrete event models to facilitate the process. Model building, facilitated or otherwise, using agent-based models, offers rich opportunities for model development and fertile ground for further research.

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