

CHAPTER 24

AGENT-BASED MODELS TO MANAGE THE COMPLEX

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This chapter introduces agent-based models and their application to management. The use of agent-based models is especially beneficial in situations where there are a number of inter-connected agents forming a complex system, be they firms, employees, customers, or other entities. We discuss the concept of agent-based modeling, building on an example model of interacting firms and customers. We discuss how this model was developed using the agent-based modeling toolkit RePast. We describe not only the advantages but also the disadvantages of agent-based models over more traditional techniques. We introduce several other agent-based models from the more general social science literature, discuss the level of complexity that should be introduced into an agent-based model, and discuss how such a technique may make an impact on the manager or organization whose strategy is to endeavor to manage the complex.

Introduction

The use of agent-based models is fast becoming an indispensable tool for studying complex systems. Over the past few years, this technique has been applied to the social sciences and more recently these models have found themselves used within mainstream management research. Their application towards business and management is relatively new, and is an area where great potential exists for studying firm behavior in complex environments.

Agent-based modeling has variously been described as the ‘third way’ of carrying out research (Gilbert & Terna, 2000), Axelrod (1997) describing agent-based modeling as differing from inductive and deductive methods, while Epstein and Axtell (1996) refer to agent-based modeling as ‘generative’ social science. But just how does this relatively new methodology differ from traditional methods? And how is it being used within organizations and management?

The terms agent-based modeling, individual-based modeling (Hiebeler, 1994), or bottom-up modeling (Pratt, *et al.*, 1993) all refer to the concept of redefining the way we model systems, differing from traditional techniques which may assume homogeneity of actors, with their focus on rationality and equilibrium, or traditional techniques that may impose rules on the system in a ‘top-down’ fashion. Agent-based modeling changes this approach. It has been suggested that agent-based modeling is a better method of modeling complex social systems, such as those encountered by managers within firms and other organizations. Whilst such approaches are being recognized in a wide range of social science disciplines, their application to mainstream management has been less prevalent. This chapter introduces the concept of agent-based modeling to business problems, while being wary of the potential pitfalls of applying the concept without critical thought or reservation.

Even though managers may be used to modeling within their corporations, the use of agent-based modeling can at the very minimum provide a new technique for modeling - one that is not predisposed to equilibrium solutions, perfect rationality, or optimization algorithms that many other models may be predicated upon.

Models and modeling within business and management

When faced with the problems of a complex world, we can try and understand them using models that we are familiar with in order to codify and analyze these problems. For example, we may rely on insight from similar situations that we have confronted in the past, or we may analyze the situation using techniques that are familiar to us, such as statistical methods. Whilst such methods may be appropriate in situations where the environment is stable and where processes are linear, such techniques can be augmented by new meth-

ologies, especially in situations of instability and nonlinearity. Agent-based modeling, which we describe in this chapter, offers the opportunity to study systems of interrelated ‘agents’ in a new way - a technique that may help our understanding of inter-firm and intra-firm processes, and may give us new insights into managerial problems that are considered too complicated to be modeled by traditional means.

Modeling as a technique is certainly not new - we can understand complicated data by using mathematical or statistical models to determine how variables interact, we may set up formalized relations between players, as in game theory, or we may use econometric models to forecast the trajectory of a system. However, each of these paradigms relies to a greater or lesser extent upon restrictions imposed by the modeling technique. For example, game theoretical approaches have been criticized for their assumptions of rational actors, and their usual limitation on the number of players (typically two) that are part of the system. Econometric models may be over-complicated, while mathematical models can typically be ‘solved’ in order to produce a closed form or analytical solution to a problem thereby being predicated on a solution being able to be found.

While agent-based modeling also relies on certain assumptions about how the model is constructed, the flexibility in the technique allows us to model a wide range of systems of inter-connected agents, as we shall demonstrate below.

Agent-based models: An example

In order to demonstrate the components of an agent-based model, we now review the construction of a typical model. We review a model constructed in *RePast* (a toolkit for constructing agent-based models) of inter-firm competition (Robertson, 2003). The basic premise of the model is that different firms compete for customers as shown in Figure 1. Each of the entities shown in Figure 1 is an *agent*.

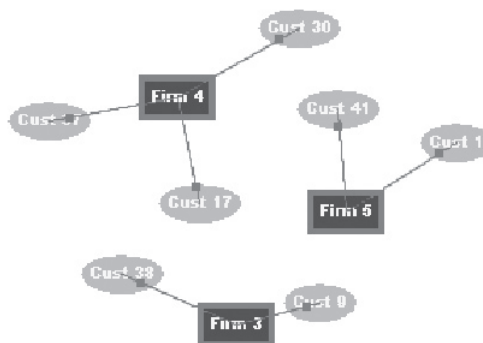


Figure 1
Firm and Customer Agents (after Robertson 2003)

Agent-based models require several components for their construction. The most fundamental components of an agent-based model are the agents themselves. Agents are the fundamental ‘building blocks’ of the system; the behavior of these agents drives the behavior of the model. In the example above, the agents include customer agents and firm agents. The linkages between agents are themselves agents, and are constructed in a similar way to the customer and firm agents.

We can think of an example of modeling firms within an industry. What agents do we need to create - to model - in order to produce an overall model of these firms? Firstly we need to define an agent type that represents the firm itself. The properties of the agent are governed by the parameters that we ascribe to an agent. In the case of a firm agent, its parameters could include the firm’s size, profitability, number of employees, and several other parameters that we may be interested in investigating. Some of the parameters that are used to specify this particular model are shown in Figure 2.

In Figure 2, we see some of the parameters that are used to construct the model. These represent the *model parameters* - those that are used to describe the initial construction of the model. For instance, we see that we can set parameters for the number of firm and customer agents within our industry, and

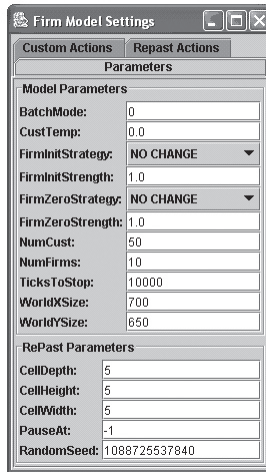


Figure 2
RePast Parameters Panel

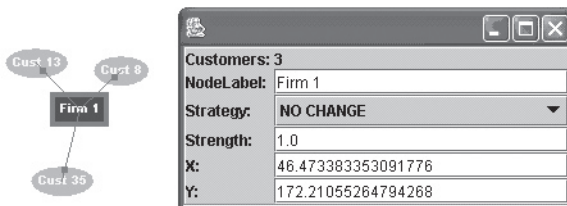


Figure 3
Probed Agent (after Robertson, 2003)

indeed how such firms move around in the industry (we specify strategies for each firm, for example a ‘no change’ strategy). However, one of the key reasons that we use agent-based modeling rather than other types of simulation is that it allows us to treat each agent as a heterogeneous, autonomous ‘being’. We are not restricted to identical agents - we can specify the behavior and parameters of individual agents (which may be different to the behavior and parameters of other agents). We can demonstrate these individual parameters by ‘probing’ an individual agent as in Figure 3.

As can be seen, each individual agent has parameters associated with that agent, for example the x and y co-ordinates of the agent, the ‘strength’ of the velocity at which it moves, and its individual strategy (in this case, the agent has a ‘no change’ strategy). Such *agent parameters* may be inherited from the model parameters, or changed as the result of a rule being executed in the model.

Agent-based models are notable in that agents can themselves be comprised of agents. For instance, the firm agents shown in Figure 1 could themselves be further comprised of other agents - for example individual employees within a firm. Such *nesting* of agents is an important phenomenon that can be achieved by the use of agent-based models. Such *sub-agents* can themselves take on heterogeneous properties that in turn contribute to the heterogeneity of the firm. Unlike other models, each agent can take on differing parameter values, thereby giving inter-agent heterogeneity. Whereas for instance in macroeconomic models, a ‘representative firm’ notion could be used (where all firms within an industry could be considered homogeneous), in an agent-based model the heterogeneity of different agents can be captured. An agent-based model could treat each individual firm or customer as a separate entity with different characteristics as defined by the parameters of that agent.

The modeler must then define how the agents interact - for instance how customers interact with firms, and how these actions influence the firms’ behaviors. This is done by means of the model itself - this defines how, at each time step, the agents change their parameters (for example their location within space). By this process, the parameters of the agents can change and evolve over time. Such *rules* of agent behavior are at the heart of the model itself: without specifying the rules of interaction of agents, the model is not fully specified. In the example above, agents each have a *strategy* that is operationalized by the rules that the agent adopts. For example, a firm agent may adopt a strategy of simply remaining still; a more sophisticated strategy would be to imitate the lead firm in the industry, which could be operationalized by determining rules that specify a movement strategy that is derived firstly by determining the number of customers of each competitor, and then moving towards the lead competitor with a speed defined by the agent parameter ‘strength’. By determining similar rules for other agent behaviors, we are able to model the strategies of the agents.

Why use agent-based models?

What are the advantages of using such an approach over traditional modeling techniques? Agent-based models differ from traditional models that look at system-wide behaviors such as systems dynamics models (Forrester, 1961). In such systems dynamics models, the unit of analysis is the macroscopic output of the system. The individuality and heterogeneity of the components flowing through the system are compromised for the macro level results.

In agent based modeling, spatial interaction between individual agents can be included in the model. Models that include a notion of proximity (whether geographical or otherwise) are well suited to agent-based modeling; each agent can have differing preferences as to when to act, and this can be determined by the proximity to other agents.

One of the most important features of agent-based models is that they need not be in equilibrium. While economic models aim to produce a 'solution', whether this be of a closed form type (i.e., by solving a set of differential or difference equations), or by finding an outcome where there is no incentive for players to change their strategies (such as discovering Nash equilibria in game theoretic models), this presumption of equilibrium is not required for agent-based models. Traditional game-theoretical models and other economic models may assume rationality of actors, whereas agent-based models are more suited to modeling boundedly rational behavior where agents adapt to their environment. Agent-based models *can* reach such states of equilibrium, for example where emergent properties are apparent where the system becomes stable over time. This is however not a requirement of an agent-based model, and in this respect, agent based models can be considered more general than equilibrium-based models.

Of course, the use of agent-based modeling will not eclipse the use of statistical, mathematical, and other types of modeling. Some of the drawbacks of using agent-based models are set out below.

Drawbacks of the agent-based paradigm

The use of agent-based modeling has disadvantages that are side-effects of its very advantages. Firstly, the models that are created do not lend themselves to analytical solution: there may be no 'solutions' to the model as there may be for a macro-economic model. Macro-economic modelers would naturally lead themselves to finding a solution whereby equilibrium is established: in the simplest models of supply and demand, this would be where the supply curve and the demand curve meet - the point of equilibrium. However, in dynamic systems that have a stochastic element, there may be no equilibrium (dynamic or otherwise), and in this regard the quest of searching for equilibrium may in fact be futile. If a model can be solved analytically, then this may be the best way of modeling a system - the results produced by an analytical model can be far more elegant than the potentially rather inelegant results from an agent-

based model (of course, agent-based models that exhibit emergence may be exceptionally elegant). However, the social and business world is inherently 'messy': we should be rather wary of basing our actions on analytical models if we assume that their results may be applied in the business world without reservation.

A further drawback of agent-based models is that it is necessary to be able to program the behavior of the agents, whether this is done via a graphical interface to software that determines the behavior of 'ready made' agents, or whether this is at a deeper level where it is necessary to include all details of the agent behavior. The use of in-depth programming requires knowledge but can enable the user to create tailored agents, whereas use of pre-defined agent types allows ease of set up of the model but with the inability to tailor the model to particular modeling requirements. While it is possible for personal computers to run models with potentially unlimited number of agents, there are practical problems that limit the number of agents that can be successfully modeled, restrictions as to the memory and processor capacities of the computer. Of course, such limitations will become less important given advances in technology, but such considerations still need to be borne in mind during the construction of the model. The limitation on the number of agents, as well as being related to the absolute number of agents required to be modeled, also depends on the level of complexity and amount of interaction between agents. If all agents act independently, this requires less resource than agents that act based on the properties of other agents, for example the behavior of an agent depending upon the states of other agents within a social network. The interrelation of agents requires more resources to be used, and consequently restricts the number of agents that can be modeled effectively.

Complexity of the agent-based model

Whereas in a statistical model there is the presumption that the more simple or parsimonious the model the better, this is not necessarily the case for an agent-based model. The level of complexity that is used in developing the model is very much dependent on the use that the model is to be put to, described by Carley (2002) as the 'veridicality' of the model. At one extreme, agent-based models can be used to model situations where the object is to 'recreate' the characteristics of the real world. Such models tend to use a large number of agents, and require significant computing power in order to create an environment that mirrors the real world.

Agent-based models fall into two broad categories - those that intend to demonstrate a particular phenomenon, particularly those that demonstrate a feature such as the emergence of a macro-level property by virtue of micro-level rules. The other group of models includes those that attempt to model reality, where the models tend to be complicated in their nature. It is important that these different types of models are distinguished and not confused; models should be explicit at their outset as to whether they purport to model the real

world or whether they are to be used for theory building. If however the model falls between these two extremes, there is potential criticism from both sides: that it does not reflect reality nor does it contribute to theory building.

Agent-based models in action

We can demonstrate several properties of agent-based models by looking at the features exhibited by some further examples of agent based models that have been used within the social sciences. One of the most interesting models within the social sciences that demonstrate such macro-level phenomena from micro-level rules is the segregation model developed by Schelling (1971, 1978). Although the model was originally demonstrated without the use of a computerized model (rather using a checkerboard and different colored counters to demonstrate the phenomenon), it is an ideal candidate to be converted into an agent-based model. The Schelling model is of interest in that it produces outcomes from the micro-level moves of the agents, so called *emergent* (Holland 1998) behavior. In Schelling's model, the agents represent people with preferences to be located near neighbors of the same color as themselves. However, even when the level of other-color tolerance is high, segregation is found to take place at the macro level, with clustering of people of different colors (Figure 4, implemented using *NetLogo*).

This result, which is not intuitive, can be investigated by the use of an agent-based model by changing the parameters of the model (for example varying the agents' tolerance for agents of a different color). The effect on the macro level properties that manifest themselves in the form of the level of segregation can then be studied.

In other examples of agent-based models, such as Epstein and Axtell's (1996) *SugarScape* model (Figure 5, implemented using *RePast*), where agents are trying to capture sugar (represented by the background color density) the rules of the agents (shown as circles in the figure) may be of the form 'look around n spaces to the north, east, south, and west, and move to the closest vacant space with the highest concentration of sugar, and collect all the sugar at that site', where n in this case would be a parameter of the agent, representing the 'vision' of that agent. A version of Epstein and Axtell's (1996) *SugarScape* model includes agents that are located within a grid, who can 'trade' with their neighbors, trading one of two goods: sugar or spice. Agents can move around the grid, either moving to collect a commodity or to trade with the agent's neighbors. Now, as each agent acts autonomously, any behavior at a macro level is made up of the interactions at the individual agent level. Epstein and Axtell (1996: 35) note that emergent properties can come out of these simple interactions, emergence being defined by them as 'stable macroscopic patterns arising from the local interactions of agents'. Such emergent properties - that can generate unexpected macro-level characteristics - are one of the most unusual properties of complex systems. Whilst previous approaches to complex behaviors such as chaos theory have been overstated as providing insights to

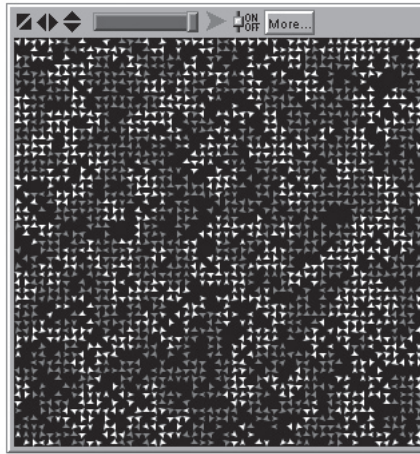


Figure 4

Model of Segregation implemented in *NetLogo* (Wilensky, 1998; after Schelling, 1971, 1978)

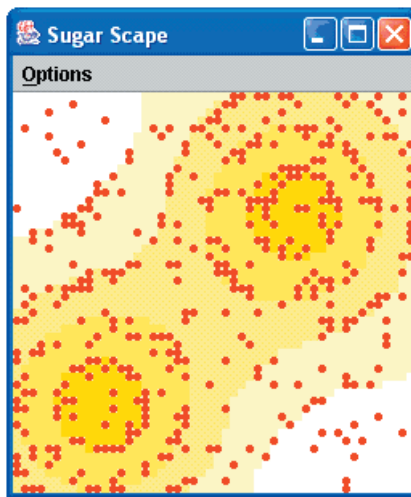


Figure 5

SugarScape Model (Collier, 2004; after Epstein & Axtell, 1996)

real systems, the complexity-based approaches to management have a greater potential (Robertson, 2004).

Most agent-based models can be visualized on some sort of space - for example a map of the geographical location of the agents, whether this is where employees are located in an organization in a geographical space or where the employees are located in abstract space, for example a space representing the employees' social network. The space in which agents move is important, as it defines the proximity of the agents to each other. If this affects the network of

how agents interact, then it is important to specify carefully the properties of the space. For instance, if the space is purely geographic, it may be important to have clearly defined boundaries (i.e., the space is bounded). However, in models such as the *SugarScape* model, it may be important to not constrain the agents, in which case the topology of a torus - a donut-shaped 'world' - may be used so that the space in which the agents are located has no boundary.

The techniques have been applied to real and theoretical situations within management. Southwest Airlines have used an agent-based model to improve the operational efficiency of airfreight routing (Seibel & Thomas, 2000); excess inventories of Proctor & Gamble have been investigated by introducing an agent-based supply chain model (Siebel & Kellam, 2003) while researchers from France Télécom are using agent-based models to simulate the interaction of the behavior of their customers (Ben Said, *et al.*, 2001), while Axelrod, *et al.* (1995) have used agent-based models to investigate the formation of alliances. On the theoretical side, agent-based models have been used to study adaptive strategies on rugged fitness landscapes (Rivkin, 2001), Rivkin's agent-based model is predicated on Kauffman's (1993, 1995) NK model to generate 'tunable' fitness landscapes. Richardson, *et al.* (this volume) discuss landscape 'metaphors' as one example of where agent-based models could be used to explore the group decision support systems of complex projects.

Simple and complicated agent-based models

The construction of agent-based models can be grouped into those that purport to present a simple model, and those that attempt to produce a complicated model which is designed to reflect reality. The first type of model, of which the Schelling model is an example, uses a simple model with relatively simple agents in order to provide an illustration of a property, in this case the emergence of macro-level segregation from the micro-level interaction between agents. The power of such models is in their ability to develop theory as opposed to offering a predictive model. Such theory, so developed, can be used for example to test empirical data and to test hypotheses generated from the model, such modelers subscribing to the kiss ('keep it simple stupid') principle (Axelrod, 1997: 5). Agent-based models can be used to demonstrate emergence (Holland, 1998) whereby macro-level properties are discovered by micro-level interactions. The Schelling result of agents grouping into segregated colors is an example of an emergent phenomenon, such an outcome being not immediately apparent from the micro-level rules that are included in the model. In some regards, we can think of emergent phenomena as being long-run equilibria of the model: in the case of the Schelling model, the segregation result can be thought of as such. It is therefore possible - in principle at least - to define analytically the conditions by which the model results in emergent characteristics being presented.

However, not all agent-based models exhibit emergent properties. One can contrast simple agent-based models with agent-based models that aim to reflect the reality of an environment. This class of models has been used to

model power grids, ecological environments, or stock markets. Real world data reflecting the structure of the environment, for example the precise interconnections between power distribution networks or the topological properties of a landscape can be incorporated into the model (for example using GIS data to recreate a geographical landscape or to incorporate every pylon and transmission wire in a power grid). With this real data being reflected accurately in the model environment, the user of the model can simulate the effects of power outages occurring in one part of the grid, this having the immediate benefit that the ability of the network to cope with an emergency can be simulated without the inconvenience and expense of testing the robustness of the system by actually exposing the system to an event such as a failure of one connection within the network. The advantage of using an agent-based model over a system wide model is that the researcher (whether they be an academic or a practitioner) may manipulate individual agents within the system to determine the effect that this individual agent has to the stability of the system. Such changes within the model may take the form of changes of parameters or rules in order to investigate whether the stability of the system is influenced by such changes. While such models are not full representations of the 'real world', their use is designed to give insight into a practical problem of relevance to managers and therefore are not 'designed' to produce macro-level properties of for example emergent behavior.

Of course, whether the model reflects the actual performance of such an event depends upon whether the model has been constructed appropriately. In order to discern whether a model is of use within an applied setting, it is important to 'dock' or calibrate such models with observed data and with other models of the same phenomenon (Axtell, *et al.*, 1996). While this is less applicable to theoretical models that are designed for theory-building, it is vital for models that are designed to reflect real world behavior.

The use of agent-based models may be particularly useful when environments in which firms operate are complicated and not able to be explained adequately by means of a traditional analysis. One of the great achievements of the community of researchers and practitioners involved in the agent-based modeling community is that models that seem applicable to, say a problem in biology (Levinthal, 1997), can potentially be transferred over to the domain of management, where it can provide an insight into managerial problems and those related to business applications.

Resources for agent-based modeling

The creation of modeling toolkits has allowed researchers to build agent-based models without the requirement of knowledge of detailed programming languages. The rise in popularity of agent-based modeling can in part be attributed to the fact that modeling resources have become easier to use. There is a rapidly growing array of toolkits available for agent-based modeling, for example the software packages *Ascape*, *NetLogo*, *RePast* and *Swarm*. However, at the

time of writing there is no generally acceptable interface that allows users to develop tailored agent-based models without at least a minimal knowledge of programming. Of course, as more users are attracted to the field, it is only a matter of time until a suitable interface can be found to allow managers to set up their own agent-based models, apply the dynamical properties of their own firm or industry, and experiment with the model to see the effects of changes in parameters or assumptions as to how the model operates. Toolkits include sample models, including the *SugarScape* and *Schelling* models, which can demonstrate some of the potential of agent-based modeling. From the point of view of enabling models to be viewed without installing computer languages, *NetLogo* is of use in allowing models to be run from within a web browser. The use of *NetLogo* is however limited by the very ease with which it can be run: more advanced models that require more complex agent behavior will find that they require toolkits such as *RePast*. *RePast* has the advantage that it allows a large amount of flexibility in construction of the agents and the agent environment. At the time of writing, the *RePast* platform appears to be the platform that enables both the flexibility of application, but also has a following in both the social science and increasingly the managerial regimes. Therefore, for business applications that require a more detailed level of sophistication, the *RePast* platform should be considered seriously to start the journey into agent-based modeling.

Conclusions

The technique of agent-based modeling has great potential, to enable managers to understand the complex world in which they operate. Potential uses are wide, and encompass models that attempt to replicate the real world, to simpler models that exhibit emergent behavior. Agent-based models should however be introduced with several caveats. Emergent properties that may be seen within a model may not be experienced in a real world situation - changes in the parameters can knock the simulation from an emergent trajectory and thus the modeled results may not transpire in reality. Furthermore, the investment in setting up the model may outweigh the potential gains that are brought about from changing the configuration of a firm to incorporate findings from the model. However, the actual process of constructing a model of the environment - specifying which agents are most important to the operation of your firm - is a worthwhile process: but are people, machines, processes, or ideas the most important agents? In reality, it may be a combination of all these and more that are drivers of profitability. We should not forget that the active process of management itself provides a tangible effect on competitiveness of the firm. Agent-based models can add to the inventory of tools available in the manager's portfolio. Whether their utilization causes beneficial effects on competitiveness and profitability depends critically on how they are designed and implemented.

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